GENERAL

piecewise functions problems and answers

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Introduction

Piecewise functions appear in nearly every branch of mathematics, from calculus and algebra to economics and engineering. Whether you're a high school student tackling a homework assignment, a college sophomore preparing for an exam, or a professional applying mathematical models to real-world data, you'll often encounter problems that require you to define a function in segments. These "piecewise functions problems and answers" are a staple of many math curricula, and mastering them can give you a solid foundation in analytical thinking and problem-solving.

In this comprehensive guide, we'll explore the fundamentals of piecewise functions, break down common problem types, walk through detailed step by step solutions, and provide a wealth of practice examples. By the end, you'll be equipped not only to solve a wide range of piecewise function problems but also to explain the reasoning behind each solution.

What Are Piecewise Functions?

A piecewise function is a function defined by multiple sub functions, each applying to a specific interval of the input domain. Instead of a single algebraic expression, the function "pieces" together several expressions, each with its own rule.

Formally, a piecewise function *f(x)* can be written as:

f(x) =
{ expression1 if condition1
expression2 if condition2
...
expressionN if conditionN }

Each condition delineates a distinct region of the domain, and the corresponding expression defines the function's value within that region. Piecewise functions are particularly useful for modeling scenarios that behave differently under varying circumstances—such as tax brackets, shipping costs, or physical phenomena that change state at specific thresholds.

Why Study Piecewise Functions?

- Real world Modeling: Many real-life processes are not continuous; they change rules at certain points. Piecewise functions capture these transitions accurately.
- Mathematical Rigor: Understanding how to construct and manipulate piecewise definitions strengthens your grasp of functions, limits, continuity, and differentiability.
- Problem Solving Skills: Working with piecewise functions trains you to analyze conditions, evaluate expressions in different intervals, and apply logical reasoning.
- Preparation for Advanced Topics: Topics such as absolute value, maximum/minimum problems, and integration over segmented domains often rely on piecewise analysis.

Common Types of Piecewise Function Problems

Below are the most frequently encountered categories of piecewise function problems in high school and early college courses. Mastery of these categories will cover the majority of exam and homework questions.

1. Graphing Piecewise Functions

Students are asked to sketch the graph of a function defined by different expressions over separate intervals. This requires understanding the behavior of each sub function and how they connect at boundary points.

2. Determining Continuity and Discontinuity

These problems involve checking whether a function is continuous at a given point. You must evaluate limits from the left and right and compare them to the function's value at that point.

3. Finding Derivatives and Slopes

When a piecewise function includes differentiable segments, you may need to compute the derivative within each interval and examine the slope at transition points.

4. Solving Inequalities Involving Piecewise Functions

These tasks ask you to find the set of all *x* values that satisfy an inequality where the function is defined in pieces.

5. Integration of Piecewise Functions

Integrating a piecewise function involves integrating each segment separately and summing the results. This is common in physics and engineering applications.

6. Optimization Problems

Optimization often requires setting up a piecewise objective function, then finding the maximum or minimum by analyzing each segment.

Step by Step Solutions for Common Problems

Below, we'll walk through representative examples of each problem type, highlighting the reasoning process and key steps.

Example 1: Graphing a Piecewise Function

Consider the function:

f(x) =
{ x^2 if x < 0
2x + 1 if 0 &le x < 3
5 if x &ge 3 }

Step 1: Identify the intervals and corresponding expressions.

Step 2: Plot each expression on its interval. For *x^2* when *x < 0*, draw a parabola opening upward, focusing on the left side. For *2x + 1*between 0 and 3, draw a straight line. For the constant 5 beyond 3, plot a horizontal line at y = 5.

Step 3: Check endpoints. Evaluate *f(0)* from the second piece: *2(0)+1 = 1*. Evaluate the limit from the left: *lim_{x!'0 {} x^2 =* 0. Since the two values differ, there is a jump discontinuity at *x = 0*. Mark an open circle at (0,0) and a closed circle at (0,1).

Step 4: Mark the point at *x = 3*. Evaluate *f(3)* from the second piece: *2(3)+1 = 7*. The limit from the left is 7, but the value from the third piece is 5. Thus, another jump at *x = 3*. Indicate an open circle at (3,7) and a closed circle at (3,5).

Result: The graph consists of a leftward parabola approaching the origin, a straight line rising to (3,7), and a horizontal line at y = 5 starting at (3,5).

Example 2: Checking Continuity

Given the piecewise function:

g(x) =
{ 3x - 2 if x < 2
  4 if x = 2
  x^2 - 1 if x > 2 }
Step 1: Compute left and right limits at *x = 2*.

Left limit: *lim_{x!'2 {} (3x - 2) = 3(2) - 2 = 4*

Right limit: *lim_{x!'2 z} (x^2 - 1) = 2^2 - 1 = 3*

Step 2: Compare limits with the function value at 2.

Function value: *g(2) = 4*.

Since the left limit equals the function value (4), but the right limit (3) differs, *g(x)* is discontinuous at *x = 2*.

Example 3: Differentiation

Find *f'(x)* for the function:

f(x) =
{ x^3 if x < 1
  2x + 3 if x &ge 1 }
Step 1: Differentiate each piece.

For *x < 1*, *f'(x) = 3x^2*.

For *x &ge 1*, *f'(x) = 2*.

Step 2: Examine the derivative at the boundary.

Left derivative at 1: *lim_{x!'1 {} 3x^2 = 3*

Right derivative at 1: *lim_{x!'1 z} 2 = 2*

Since the left and right derivatives differ, the function has a corner at *x = 1*, so *f'(1)* does not exist.

Example 4: Solving an Inequality

Solve *f(x) < 0* for:

f(x) =
{ x - 4 if x < 0
  -x + 2 if x &ge 0 }
Step 1: Solve each piece separately.

For *x < 0*, *x - 4 < 0* !*x < 4*. Since *x < 0*, this condition is always satisfied. Therefore, all *x < 0*satisfy the inequality.

For *x &ge 0*, *-x + 2 < 0* !-*x < -2* !*x > 2*. Combine with *x &ge 0*, we get *x > 2*.

Result: The solution set is *(-" , 0) "* (2, ")*

Example 5: Integration

Evaluate *"+_0^4 f(x) dx*where:

f(x) =
{ 3 if 0 "d x
Step 1: Split the integral at the boundary point 2.

"+_0^4 f(x) dx = "+_0^2 3 dx + "+_2^4 (2x - 1) dx.

Step 2: Compute each part.

First part: 3x |0^2 = 3(2) - 0 = 6.

Second part: "+ (2x - 1) dx = x^2 - x. Evaluate from 2 to 4: (4^2 - 4) - (2^2 - 2) = (16 - 4) - (4 - 2) = 12 - 2 = 10.

Result: Total integral = 6 + 10 = 16.

Example 6: Optimization

A company sells two products, A and B. The profit per unit for each product depends on the total units sold, described by:

P(x) =
{ 5x if 0 "d x "d 100
500 - 2x if 100
Find the number of units *x* that maximizes total profit.

Step 1: Analyze each segment.

Segment 1 (*0 "d x "d 100*): P(x) = 5x is increasing, so maximum at *x = 100* gives P = 500.

Segment 2 (*100 < x "d 200*): P(x) = 500 - 2x is decreasing, so maximum at the left endpoint *x = 101*gives P = 500 - 202 = 298.

Result: The maximum profit occurs at *x = 100* units, yielding a profit of $500.

Practice Problems with Answers

Test your understanding with the following problems. Answers are provided immediately after each question.

Problem 1

Graph the function:

h(x) =

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